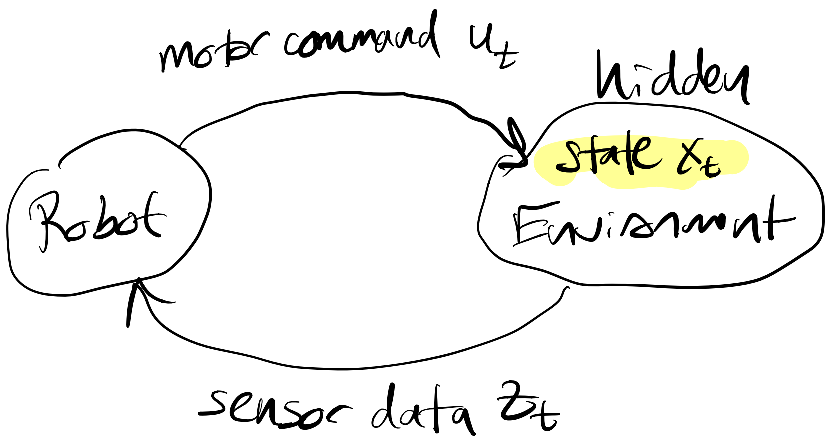
Day 1 : conceptual overview

Night 1 : mathematical foundations

Day 2 : derivation

Diagram



Timeline

robot starts in some state x_0
executes command u_1
arrives in state x_1
receives sensor data z_1
execute command u_2
arrive in state x_2
receive sensor data z_2

time

Goal

given

$$p(x_t \mid u_1, u_2, \dots u_t, z_1, z_2 \dots z_t)$$

want to know   we can observe

Reminder of tools

Bayes' Rule: $p(A|B) = \dfrac{p(B|A)\,p(A)}{p(B)}$

$$p(A|B,C) = \frac{p(B|A,C)\,p(A|C)}{p(B|C)} \quad *$$

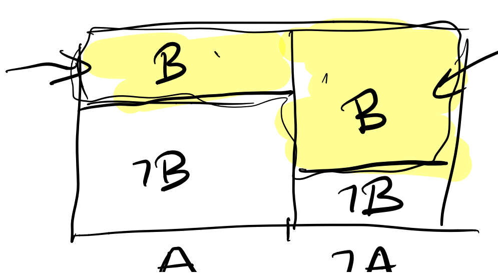
Product Rule:

$$\underbrace{p(A,B)}_{\text{A and B}} = \underbrace{p(A)}_{\text{A first}}\underbrace{p(B|A)}_{\text{B given A}}$$

Sum Rule:

$$p(B) = p(A,B) + p(\neg A, B)$$

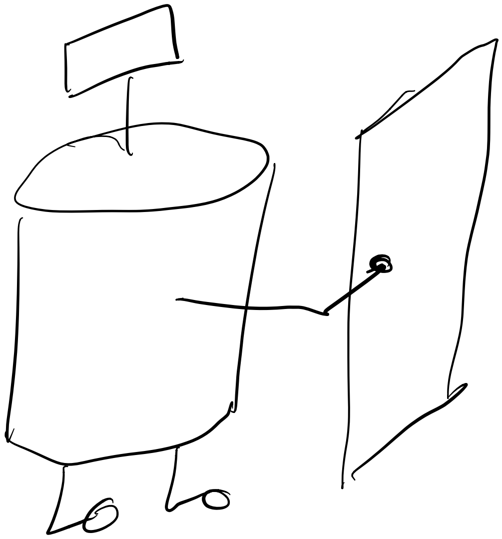
$p(A,B)$



$p(\neg A, B)$

Example

$$x_t = \begin{cases} 1 & \text{if door open} \\ 0 & \text{" " closed} \end{cases}$$



$$u_t = \begin{cases} 1 & \text{if robot pushes arm} \\ 0 & \text{" " doesn't push} \end{cases}$$

$$z_t = \begin{cases} 1 & \text{if it senses door open} \\ 0 & \text{" " " " closed} \end{cases}$$

$$\boxed{\text{Goal}: \ p(x_t \mid u_1 \ldots u_t, z_1 \ldots z_t)}$$

We need a model! $p(x_0=1)+p(x_0=1)=1$

(1) $p(x_0=1) = 1/2$, $p(x_0=0) = 1/2$

$p(z_t=0 \mid x_t=1) = 2/5$

(2) $p(z_t=1 \mid x_t=1) = 3/5$ } sensor model

$+$ $p(z_t=1 \mid x_t=0) = 1/5$

$\neq 1$ necessarily

(3) $p(x_t=1 \mid x_{t-1}=0, u_t=0) = 0$

$p(x_t=1 \mid x_{t-1}=1, u_t=0) = 1$ } motor model

$$p(x_t=1 \mid x_{t-1}=0, u_t=1) = 4/5$$
$$p(x_t=1 \mid x_{t-1}=1, u_t=1) = 1$$

Problem 1

$$p(x_1=1 \mid z_1=1, u_1=0)_{3/5}$$

sensor model

$$= \frac{p(z_1=1 \mid x_1=1, \cancel{u_1=0})\, p(x_1=1 \mid u_1=0)}{p(z_1=1 \mid u_1=0)}$$

$$\underline{p(x_1=1 \mid u_1=0)} = p(x_1=1, \underline{x_0=0} \mid u_1=0) + p(x_1=1, \underline{x_0=1} \mid u_1=0)$$

$$= \overset{1/2}{p(x_0=0 \mid \cancel{u_1=0})}\, p(x_1=1 \mid x_0=0, u_1=0) + \overset{1/2}{p(x_0=1 \mid \cancel{u_1=0})}\, p(x_1=1 \mid x_0=1, u_1=0)$$

$$= 1/2$$

$$p(x_1=1 \mid z_1=1, u_1=0) = \frac{(3/5)(1/2)}{p(z_1=1 \mid u_1=0) = N}$$

$$P(x_1=0 \mid z_1=1, u_1=0) = \frac{\overset{1/5}{\cancel{P(z_1=1 \mid x_1=0, u_1=0)}}\, P(x_1=0 \mid u_1=0)}{\cancel{P(z_1=1 \mid u_1=0)}\ N}$$

$$P(x_1=0 \mid u_1=0) = P(x_1=0, x_0=0 \mid u_1=0)$$

$$+ P(x_1=0, x_0=1 \mid u_1=0)$$

$$= \overset{1/2}{\cancel{P(x_0=0 \mid u_1=0)}}\ \overset{1}{\cancel{P(x_1=0 \mid x_0=0, u_1=0)}}$$

$$+ \overset{1/2}{P(x_0=1 \mid \cancel{u_1=0})}\ \overset{0}{\cancel{P(x_1=0 \mid x_0=1, u_1=0)}}$$

$$= \underline{1/2}$$

$$P(x_1=0 \mid z_1=1, u_1=0) = \boxed{\frac{(1/5)(1/2)}{\boxed{N}}}$$

$$P(x_1=0 \mid z_1=1, u_1=0) + P(x_1=1 \mid z_1=1, u_1=0)$$

$$= 1$$

$$\frac{(1/5)(1/2)}{N} + \frac{(3/5)(1/2)}{N} = 1$$

$$N = (1/5)(1/2) + (3/5)(1/2) = 2/5$$

$$\Rightarrow P(x_1=1 \mid u_1=0, z_1=1) = \frac{(3/5)(1/2)}{(2/5)} = 3/4$$

Problem 2

swap w/ Bayes!

$$P(X_2=1 \mid \underline{z_1=1}, \underline{z_2=1}, \underline{u_1=0}, \underline{u_2=1})$$

$$= \frac{\overbrace{P(\cancel{z_2=1 \mid} X_2=1, \cancel{u_1=0, u_2=1})}^{3/5} \times \cancel{+} P(X_2=1 \mid z_1=1, u_1=0, u_2=1)}{\cancel{P(z_2=1 \mid z_1=1, u_1=0, u_2=1)}\ N}$$

$$P(x_2=1 \mid z_1=1, u_1=0, u_2=1) =$$

$$P(x_2=1, \boxed{x_1=0} \mid z_1=1, u_1=0, u_2=1) +$$

$$P(x_2=1, \boxed{x_1=1} \mid z_1=1, u_1=0, u_2=1)$$

$$= \boxed{P(x_1=0 \mid z_1=1, u_1=0, \cancel{u_2=1})}$$ motor model

$$\times P(x_2=1 \mid x_1=0, \ldots$$

$$p(x_2=1 \mid x_1=0, \cancel{z_1=1}, u_1=0, \cancel{u_2=1})$$

$$+ \boxed{p(x_1=1 \mid z_1=1, u_1=0,} \cancel{u_2=1})$$

$$\times p(X_2=1 \mid X_1=1, \cancel{z_1=1}, u_1=0, \cancel{u_2=1})$$

Bayes Binary Filter

$$p(x_t=1 \mid u_1 \ldots u_t, z_1 \ldots z_t) \propto$$

$$\underline{p(z_t \mid x_t=1)} \times \{ \boxed{p(x_{t-1}=0 \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})}$$

$$\times p(x_t=1 \mid x_{t-1}=0, u_t)$$

$$+ p(x_{t-1}=1 \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})$$

$$\times p(x_t=1 \mid x_{t-1}=1, u_t) \}$$

Bayes Discrete Filter

$$p(x_t=i \mid u_1 \ldots u_t, z_1 \ldots z_t)$$

$$\stackrel{\alpha}{=} p(z_t \mid x_t=i) \sum_{j=1} p(x_{t-1}=j \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})$$

$$* p(x_t=i \mid x_{t-1}=j, u_t)$$

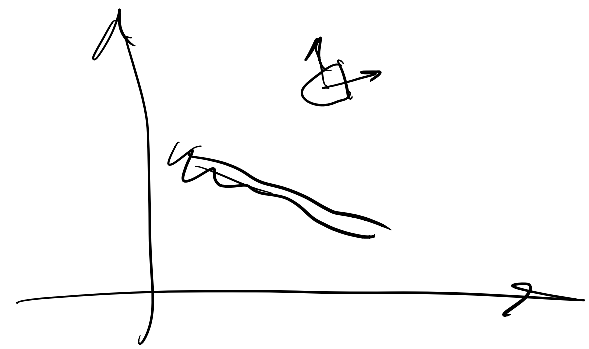
*

Scalability: suppose I have K states

$O(K)$

$\Rightarrow O(K^2)$

Neato



X_t = position and orientation of Neato

2 position dimensions
1 angle orientation
$\}$ 3d

x	y	θ	X_t
0	0	0	1
0	0	0.1	2
0	0	0.2	3

$\vdots$

100 choices x

" " y

" " θ

1,000,000

updates would take 1,000,000,000,000

Particle Filter

Particle i at time t written as $X_t^{(i)}$

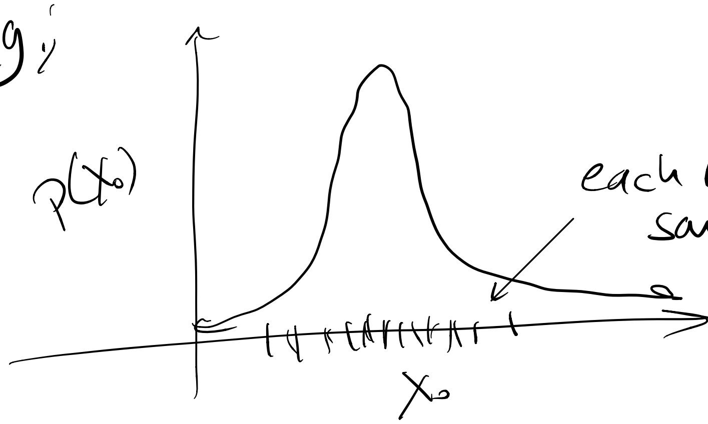
Given:

- Initial state model $p(X_0)$
- Motor model $p(X_t \mid X_{t-1}, u_t)$
- Sensor model $p(Z_t \mid X_t)$

Step 1: Create initial particle set

For i from 1 to m: $X_i^{(0)} \sim p(X_0)$ (note: "$\sim$" means sample from)

e.g.;



Step 2: apply motion model

For i from 1 to m:

$$\tilde{x}_i^{(t)} \sim p(x_t \mid x_{t-1} = x_i^{(t)}, u_t)$$

Step 3:

Compute weights

$$w_i^{(t)} = \underbrace{\frac{p(z_t \mid x_t = \tilde{x}_i^{(t)})}{\sum_{j=1}^{m} p(z_t \mid x_t = \tilde{x}_j^{(t)})}}_{\text{ensures weights sum to 1}}$$

Step 4

Resample particles

For i from 1 to m:

$$\underbrace{p(x_i^{(t+1)} = \tilde{x}_j^{(t)})}_{\text{copy each particle with probability equal to its weight.}} = w_j^t$$

Step 5: go to Step 2!